# Collective search for information

–take 2–

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## 1 The problem

Assume that a product is characterized by three different dimensions (or characteristics) $A$, $B$ and $C$, so that its final value is determined by a 3D-function such that

$$V : \mathbb{R}^3 \to \mathbb{R}, \tag{1}$$

$$(v^A, v^B, v^C) \mapsto V(v^A, v^B, v^C) \overset{(*)}{=} v^A + v^B + v^C. \quad \text{(*) for simplicity} \tag{2}$$

Assume also that the product is distributed in a two-dimensional space $S \subset \mathbb{R}^2$ (for example, a square). Then, each function value $v^X$, $X = A, B, C$, is such that

$$v^X : S \subset \mathbb{R}^2 \to \mathbb{R}, \tag{3}$$

$$(x, y) \mapsto v^X(x, y). \tag{4}$$

Assume finally that the space $S$ is divided in a grid of $N_c^x \times N_c^y$ cells, and denote by $v_{i,j}^X$ the (average) value of the function $v^X(x, y)$ on the cell $C_{i,j}$; see Fig. 1.

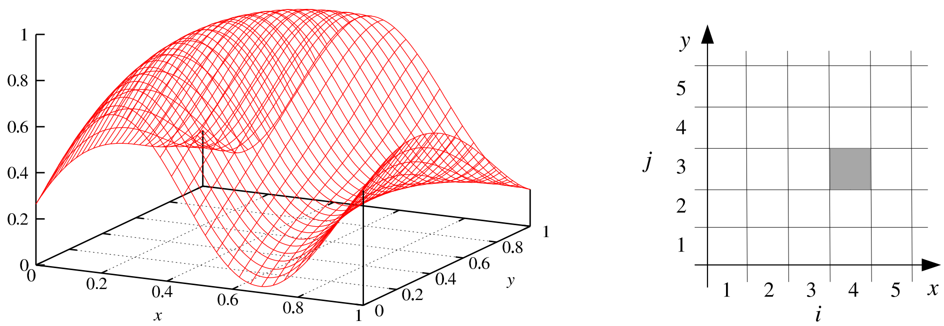


Figure 1: Left: Simple value function $v^X(x, y)$ of the component $X$ of the product. Right: Spatial grid that players have to explore, with the cell $C_{4,3}$ highlighted.

The problem consists in finding the cell(s) with the maximum value of the product $\max(V)$. Players don't know the function $V(x, y)$ and will have to explore the field according to some rules. They will have access to some partial information –partial both spacially and qualitatively– that they will have to share (but they can eventually decide not to share the information they obtain individually).

We propose to analyse this problem by means of **a sequential game**.

In each round, a player is able to explore a limited number of cells. Each player is sensitive to only one characteristic (dimension) of the product, so that the information he collects is partial. Then, the player decides to what extend he wants to share this information with the other players. The mechanism of information transfer is what introduces the collective component of the model: players have acces to both the information of the dimension of the product they are sensitive to, and the information provided by previous players. We expect that the objective will be found *by the group* by combining the specific individual knowledge that each kind of player can obtain about the components of the product with the qualitative information they build collectively.

# 2 Sequential game

## 2.1 Elements of the game

The game is played sequentially in rounds, with one player per round.

- The players: Let $N$ denote the number of players. Each player can only detect one of the three dimensions of the product, $A$, $B$ or $C$. We thus have three types of player; we assume (for simplicity, at the beginning) that there is $N/3$ players of each type.
- The field: The product is distributed in $N_c$ cells of the same size, equally accessible to all players, where the value is not homogeneously distributed. The field we consider in our example is a two-dimensional field but cells can be numbered with a single index $j = 1, \ldots, N_c$ (we can also consider one-dimensional fields...).
- The objective: The simplest collective objective is to find the cell containing the maximum value of the product, that is, the cell $C_k$ such that

$$V_k = V(v_k^A, v_k^B, v_k^C) = \max_{j=1,\ldots,N_c} \{V_j = V(v_j^A, v_j^B, v_j^C)\}. \quad (5)$$

  In a second stage, we can consider secondary objectives such as optimizing the time spent in finding these cells, or the total cost at the collective level, or a balance of both, etc.

If all players have access to all the cells and there is a perfect transfer of information between players, then the objective is easily attainable. Here we are interested in the case where 1) players have a limited access to the cells, and 2) information transfer is not perfect.

## 2.2 Access to information. Sharing information. Pheromone field

A single individual cannot achieve the objective by himself because he is blind to two of the three dimensions. Thus, information must necessarily be shared.

### 2.2.1 Two kinds of information: the value component $v_{i,j}^X$ and the pheromone field $f_{i,j}(t)$

We propose to use an analogy with ants, where information transfer takes place through a mechanism based on pheromones

When a player of type $X$ explores a cell, he receives the information relative to the dimension $X$ of the product, that is, $v_{i,j}^X$; see the upper panel in Fig. 2.

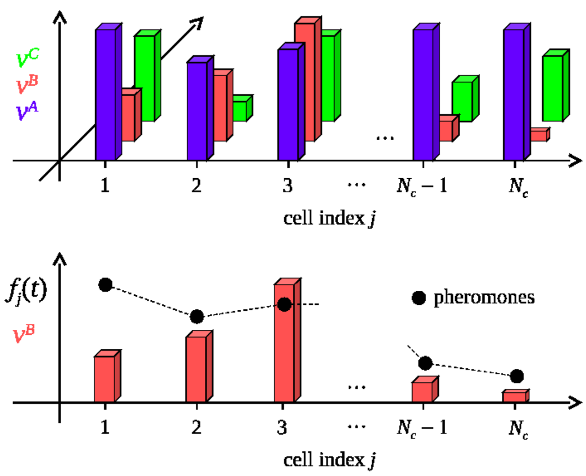


Figure 2: Top: the whole information: the three components (here colors) of the value of the product. Bottom: What a player sees: one of the three dimensions, and the instantaneous field of pheromones.

According to some rules, the player can decide to deposite a certain number of *units of information* –the pheromones– in this cell. This number can be zero. Another option is that the player is forced to deposite a fixed number of units (say, 3 or 4) per round, and another one is that the player has to distribute the units hierarchically: 3 units in one cell, 2 units in another cell, and 1 unit in a third cell.

After some rounds, different cells can contain a different number of units of pheromones; let $f_j(t)$ denote the instantaneous amount of pheromone that the cell $C_j$ contains. Here time means "number of rounds". We will refer to $f(t)$ as *the pheromone field at round or at time* $t$.

There are then two qualitatively different sources of information available for a player in each round: the information about the value of the dimension of the product the player can detect, $v_j^X$, and the information provided by the amount of pheromone found in the cell, $f_j(t)$. The first type of information is a kind of private information, in the sense that only players of one type have access to a given dimension of the product, while the information of the pheromone field is accessible to all players.

Note also that the value of the product is fixed at the beginning of the game and remains constant along the game (variation in time of one or several values $v^X$ can be introduced later), while the amount of pheromone in a cell can change in each round.

### 2.2.2 Information transfer mechanism

The mechanism of information transfer consists of two parts:

(*i*) Information collection: In each round, a player has access to only 3 cells. From each cell, he receives the information related to the dimension of the product he is able to detect, and the information of the amount of pheromones present in the cell. If the player is of type $A$, then the information he receives is $v_j^A$, $v_k^A$ and $v_l^A$ on the one hand, and $f_j(t)$, $f_k(t)$ and $f_l(t)$ on the other hand, where $j$, $k$ and $l$ are the cells the player has visited.

(*ii*) Information processing and decision making: After considering all this information, the player chooses to deposite or not a certain amount of pheromone in each one of the 3 cells. This is the part where the player adds information for the following players. We suggest that, in each round, the player can deposite a limited number of up to 4 units of pheromones. At the beginning we will consider that all combinations are possible: 4-0-0, 3-1-0, 2-1-1, but also 2-1-0, 1-0-0 or even 0-0-0.

The number of visited cells (here 3) is of course a parameter of the problem, but we guess that it will be somehow related to the size of the group $N$, the number of rounds, the velocity of convergence, etc.

Special conditions can be imposed on the cells that a player can explore, *e.g.*, cells can be required to be spatially consecutive or not too far from the the cells explored in previous rounds of the same player, a cell can be revisited by the same player, in order to deposite more pheromones and reinforce the information about this cell, or on the contrary, revisitation can be forbidden (thus introducing a memory effect in the behavior of the player), etc.

### 2.2.3 Pheromone evaporation

A key ingredient of the pheromone analogy is that the number of pheromone units that are present in a cell $f_j(t)$ can decrease in time. In ants, pheromones have a limited time of permanence due to chemical evaporation (pheromones are a chemical substance). Under the same general conditions (density of ants, transitability of the field, etc.), short paths are more intensively reinforced than long paths because short paths are more frequently travelled (positive feedback). Moreover, suboptimal paths (with respect to an optimal path) are erased from the searching strategy, allowing the colony (*i.e.*, the ants at the collective level) to finally select the shortest path.

Our idea is thus to mimic this mechanism by introducing an evaporation factor $\alpha_0$ that makes the number of units of pheromone to decrease in each round (or each $N_e$ rounds).

A first suggestion is that, in each round (*i.e.*, $N_e = 1$), $f_j(t)$ decreases according to

$$\forall j = 1, \ldots, N_c : \; f_j(t+1) = \alpha_0 f_j(t), \tag{6}$$

with $\alpha_0 < 1$, for example $\alpha_0 = 0.9$. We anticipate that this factor, that determines the meanlife of a pheromone, will be an important parameter of the model.

Alternatively, we can use an evaporation function $\alpha(f)$, so that $f_j(t+1) = f_j(t) - \alpha(f_j(t))$. A simple example of evaporation function is

$$\alpha(f) = \begin{cases} \alpha_0 \text{ units of pheromones} & \text{if } f > 0, \\ 0 & \text{if } f = 0, \end{cases} \tag{7}$$

with $\alpha_0 = 0.1$. The values of $\alpha_0$ in these examples are taken assuming that a pheromone unit has a value of order 1 (in fact, the simplest case is with 1).

## 2.3 The cost and the benefit

The number of units of pheromones deposited by the player in one round can be viewed as the *cost* of the information. When thinking about playing the game with humans, we suggest to think about the units of pheromones as coins (10 cent euro coins!).

• How players obtain these units of pheromones? Initially each player starts the game with a certain amount of units of pheromone, $N_f$, which can be zero. Then, in each round, the player receives a number of units that he can deposite in the cells he has explored in this round or that he can save for next rounds, according to the rules of the specific game. There are several possibilities (see above).

The cost is payed individually by the players, round after round. In turn, the *benefit* can revert at both levels, the individual and the group. The benefit is also calculated in terms of the number of units of pheromones. The collective benefit should be proportional to the amount of information the group has at the collective level. The fact that obtaining more information has a higher cost constitutes the trade-off of the game.

• How to determine the benefit? Think again about the ants, and the colony. At the collective level, after a number of $t$ rounds, the colony has a knowledge about the distribution of the value of the product. One expect that ants will retrieve the product from the cells where the density of pheromones is higher, so that the collective benefit can be evaluated as follows:

$$R(t) = \sum_{j=1}^{N_c} f_j(t)\, V_j(v_j^A, v_j^B, v_j^C), \tag{8}$$

that is, the sum of the value of the cells that have been visited at least once, factor (*i.e.*, proportionally to) the amount of pheromones deposited in the cell.

At the individual level, the player (the ant) can obtain a benefit in two ways. First, the information obtained at the collective level (the colony) can be redistributed homogeneously among the members of the colony, so that each player receives $R(t)/N$. This can be done instantaneously or each $N_r$ rounds. Second, each player can receives an instantaneous individual benefit for each round he plays, *e.g.*, the value of the cell he has acceded to during this round (with or without a factor proportional to the units of pheromones there is in the cell or he has deposited):

$$r_i(t) = \alpha_j V_j f_j(t) + \alpha_k V_k f_k(t) + \alpha_l V_l f_l(t), \tag{9}$$

where $\alpha_j$ is the amount of pheromones the player has deposited in cell $C_j$ during the last round. We suggest to use a combination of both.

The collective objective can thus be reformulated as

$$\text{maximize} \left( \sigma_{\text{coll}} R(t) + \sigma_{\text{indiv}} \sum_{i,t}^{N,T} r_i(t) \right), \tag{10}$$

where $\sum$ is taken over all players and along a given interval of time (a number of rounds).

## 3 The game

Give values to $N$, $N_f$, $N_r$, $N_c$, $N_e$ and $\{v_j^X\}_{j=1,\ldots,N}^{X=A,B,C}\ldots$ and repitedly follow these steps:

1. Choose randomly a player $P$ (sampling with repetition). The player will be sensistive to one dimension of the product. Denote him by $P_i^X$.
2. The player chooses 3 cells $j$, $k$, $l$, and receives the information related to these cells, $v_s^X$ and $f_s(t)$, for $s = j$, $k$ and $l$. Optionally, he receives some units of pheromones, according to $r_i(t)$. Initially $f_j(t) = f_0 = 0$, for $j = 1, \ldots, N$, for simplicity.
3. The player decides where and how to allocate his 3 or 4 units of pheromones (or 10 cent coins).
4. The number of pheromones is updated at the local level in the cells visited in the actual round, $f_s(t)$, for $s = j, k, l$, and, at the global level (the whole field of pheromones), is evaporated according to $f_j(t+1) = \alpha_0 f_j(t)$ or $f_j(t+1) = \alpha(f_j(t))$, for $j = 1, \ldots, Nc$.
5. Individual and collective benefits are evaluated and redistributed, according to $R(t)$, $r_i(t)$, $N_r$,...
6. $t \to t+1$ and `goto 1`.

A criterion to stop the game can be to evaluate the difference between the pheromone field $f[x,y](t)$ and the value function $V(x,y)$. If $V(x,y)$ is sufficiently smooth, then the maxima of $f(t)$ should be close to those of $V(x,y)$. We are then interested in reducing this distance and the time (number of rounds) required to reach a sufficiently close value.

$$d\left(\left(x_V^M, y_V^M\right), \left(x_f^M, y_f^M\right)\right), \tag{11}$$

where $\left(x_V^M, y_V^M\right)$ and $\left(x_f^M(t), y_f^M(t)\right)$ are such that

$$V\left(x_V^M, y_V^M\right) = \max_{(x,y)\in S}\{V(x,y)\}, \quad f\left(x_f^M(t), y_f^M(t)\right) = \max_{(x,y)\in S}\{f[x,y](t)\}. \tag{12}$$

## 4 Options

Players can receive a number of coins each round they play. Then he can decide if he wants to save all the coins for him and simply take profit about the information provided by the others (through the pheromone field), thus adopting a kind of defecting strategy. It will be interesting to observe the effect of the proportion of “defectors” in the survival of the colony.